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Optical multistability and Zeeman degenerate transitions

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A variety of system behaviours appears possible when $J_1 = 1 \leftrightarrow J_u = 0$ atoms interact with σ^+ and σ^- radiation modes in a cavity. Certain choices of the two ground-level collision rates allow asymmetry to develop between the output σ^+ and σ^- fields, as is also found for $J = \frac{1}{2} \leftrightarrow J = \frac{1}{2}$ atoms. However, this possibility is excluded when physical requirements are made on the rates.

Introduction

One expects that increasing the complexity of the model atom in an optical cavity will result in system behaviour more complex than the simple bistable behaviour associated with a two-state atom. A three-state (lambda) model atom, for example, can mediate a competitive interaction between σ^+ and σ^- radiation modes, as shown by Kitano *et al.* (1981). Subsequent improvements in the treatment of this model (see, for example, Savage *et al.* 1982; Arecchi *et al.* 1983) have resulted in the prediction of a variety of interesting behaviours. Real atomic transitions have at least four states, which are coupled by radiation and collisions in gas phase. Thus the validity of simple atom models in optical bistability, especially in description of experiment, must be questioned. It has been shown (Ballagh *et al.* 1981) that if the cavity radiation is constrained to a pure polarization, a limited number of dipole transitions reproduce the 'two-state' form of macroscopic dipole. Saturation fields, though, are altered by the effect of optical pumping into radiatively inactive states.

When the system is free to determine its own polarization, its behaviour becomes more dependent on the choice of atom. The $J = \frac{1}{2} \leftrightarrow J = \frac{1}{2}$ atom in a ring cavity has been studied in detail by Hamilton *et al.* (1982), who confirm that an input field with equal amplitudes of σ^+ and σ^- radiation may produce an output in which one of the modes is dominant. The $J = \frac{1}{2}$ atom, however, does not develop ground level coherence and thus it is not an accurate realization of the lambda model atom. The simplest candidate for 'lambda' behaviour is an atom with lower level angular momentum $J_1 = 1$ and upper level angular momentum $J_u = 0$. In this paper we examine the system behaviour of such atoms interacting resonantly (absorptively) in a ring cavity with σ^+ and σ^- radiation modes.

Mean field solution for $J_1 = 1 \leftrightarrow J_u = 0$ atoms in a ring cavity

The real amplitudes $X_\pm$ of the $\sigma^\pm$ plane wave cavity fields are written in units of a saturation field $\hbar\{3\gamma\Gamma_1(\mathrm{lu})\}^{\frac{1}{2}}/d$. Here γ is the spontaneous decay rate, $\Gamma_1(\mathrm{lu})$ is the decay rate of the optical dipole coherence and d is the reduced dipole matrix element. In an irreducible representation (Omont 1977) the relevant atomic density matrix elements evolve according to a set of ten coupled equations and give rise to an absorption $\frac{1}{2}\alpha\eta_{\pm}$ for $X_{\pm}$. Here α is the weak field absorption coefficient,

$$\eta_{\pm} = \{1+\beta_1(X_+^2+X_-^2)\pm(\beta_2-\beta_1)(X_+^2-X_-^2)\}/D, \tag{1}$$

$$D = \{1+\beta_1(X_+^2+X_-^2)\}\{1+\tfrac{4}{3}(2+\beta_2)(X_+^2+X_-^2)\}+\tfrac{1}{3}(8+\beta_2)(\beta_2-\beta_1)(X_+^2-X_-^2)^2, \tag{2}$$

and the ground level collisional relaxation rates, Γ_1 for orientation ($K = 1$ multipole) and Γ_2 for alignment ($K = 2$), appear in β_1 (equals γ/Γ_1) and β_2 (equals γ/Γ_2). The steady-state mean field solutions for the ring cavity system obey the coupled equations

$$Y_{\pm} = X_{\pm}(1+2C\eta_{\pm}), \tag{3}$$

where $Y_{\pm}$ are the fields incident upon the cavity (scaled by mirror transmittance) and C is the usual cooperativity parameter. By specifying the input polarization $\xi = Y_+/Y_-$ and eliminating Y_+, Y_- from (3), a fifth-order polynomial in X_+ (with coefficients in X_-) is obtained. The solution of the polynomial leads to system curves (Y_+, X_+, X_-), which can be mapped as a projection on the (Y_+, X_+) (or X_+, X_-) plane.

Linear input polarization

The most interesting behaviour occurs when $\xi = 1$, so that neither input mode is initially favoured. Subtracting the equations in (3) leads, with the transformation $u = X_+^2+X_-^2$ and $v = 2X_+X_-$, to the state equation

$$(X_- - X_+)\,[\beta_2\{\beta_1+\tfrac{1}{3}(8+\beta_2)\}u^2+\{\beta_1+\tfrac{1}{3}(8+4\beta_2)+2C\}u+(1+2C)$$
$$+2C(\beta_2-\beta_1)v-\tfrac{1}{3}(8+\beta_2)(\beta_2-\beta_1)v^2] = 0. \tag{4}$$

A symmetric output $(X_+ = X_-)$ is always present, which gives the familiar optical bistability state equation $2^{\frac{1}{2}}Y_+ = u^{\frac{1}{2}}[1+2C\{1+\frac{4}{3}(2+\beta_2)u\}^{-1}]$, where the factor $\frac{4}{3}(2+\beta_2)$ shows the effect of optical pumping into the $m = 0$ ground state.

An asymmetric branch $(X_+ \neq X_-)$ may also appear, and is described by the u, v polynomial in (4). The coefficients of u and u^2 are positive, hence only one real solution for u and only one asymmetric branch (but with degeneracy $X_+ \leftrightarrow X_-$) exist. This branch forms a simple closed loop in (Y_+, X_+, X_-) space and crosses the symmetric branch at two bifurcation points (B_1 and B_2 in figure 1) found by setting $u = v$. The existence condition for the asymmetric branch is that these bifurcation points be real and positive and from the quadratic in u((4) with $v = u$) we get the requirement

$$\{\beta_1(1-2C)+4\beta_2(\tfrac{1}{3}+C)+\tfrac{8}{3}\}^2 > \tfrac{16}{3}\beta_1(2+\beta_2)(1+2C) \tag{5}$$

or less strictly, but necessarily, $C > \frac{1}{2}$ and $\beta_1 > 2\beta_2$. Unstable parts of the curve are indicated by broken lines, and on the symmetric branch occur between turning points and between bifurcation points. In the régime $\beta_2 \ll 1 \ll \beta_1$ (figure 1*a*) the system's initially symmetric output switches abruptly at B_1 to asymmetric output $(X_+ > X_-)$ at E, and reverts to symmetric output at F. The similarity to the $J = \frac{1}{2}$ behaviour can be understood from the similarity in this régime of the (u, v) polynomial (4) to the polynomial describing the $J = \frac{1}{2}$ asymmetric branch (Hamilton *et al.* 1982). In figure 1*b* ($\beta_1 = 100$, $\beta_2 = 10$) a type of behaviour that does not occur for $J = \frac{1}{2}$ appears in the region near B_2, where no stable outputs exist, indicating the possibility of oscillation.

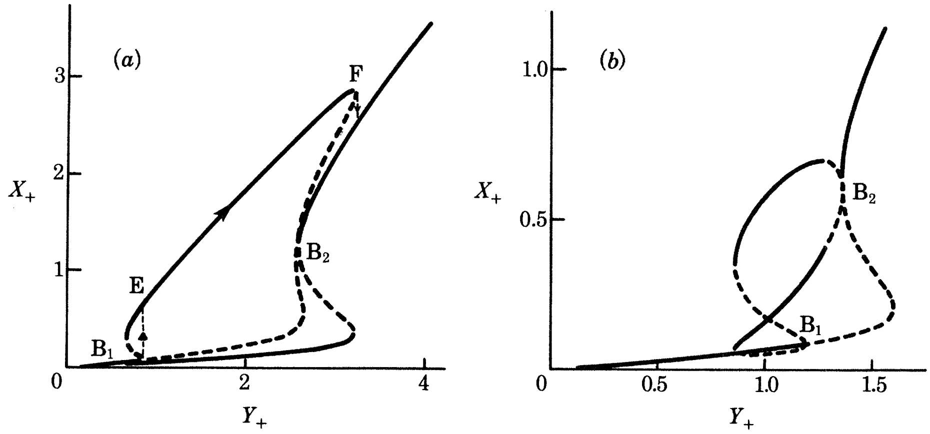

FIGURE 1. Input–output curves for $\xi = 1$ and (*a*) $C = 5$, $\beta_1 = 100$, $\beta_2 = 0.01$; (*b*) $C = 8$, $\beta_1 = 100$, $\beta_2 = 10$.

INTERPRETATION

Asymmetric output is produced when a (fluctuation generated) difference between the σ^+ and σ^- absorptions can be sustained, thus altering the relative intensities of the two cavity fields. Absorption, given (for σ^-) by

$$\eta_- = \rho_{1,1}(\mathrm{l}) - \rho_{0,0}(\mathrm{u}) + \frac{X_+}{X_-}\rho^2_{-2}(\mathrm{l}), \tag{6}$$

depends on the population difference between the states of the transition $(\rho_{1,1}(\mathrm{l}) - \rho_{0,0}(\mathrm{u}))$ and on the ground level coherence between the $m = \pm 1$ states (the alignment $\rho^2_{-2}(\mathrm{l}) = \rho_{1,-1}(\mathrm{l})$ in Zeeman representation). This coherence depends most strongly on the collisional decay rate Γ_2, since

$$\rho^2_{-2}(\mathrm{l}) = -2\beta_2 X_- X_+\{1+\beta_1(X_+^2+X_-^2)\}/3D. \tag{7}$$

Thus if β_2 is small, the difference between σ^- and σ^+ absorption is given by the difference between $m = 1$ and $m = -1$ populations, i.e. the orientation

$$\rho_{1,1}(\mathrm{l}) - \rho_{-1,-1}(\mathrm{l}) \equiv 2^{\frac{1}{2}}\rho^1_0(\mathrm{l}) = 2\beta_1(X_+^2 - X_-^2)\{1+\beta_2(X_+^2+X_-^2)\}/3D. \tag{8}$$

This equation shows that when $X_+ > X_-$, population is pumped from $m = -1$ into $m = 1$, and is not rapidly equilibrated if Γ_1 is small (β_1 large). Thus the weaker X_- radiation is more strongly absorbed and, given sufficient cavity feedback, the X_+ radiation becomes dominant.

In the régime $\beta_2 \gg \beta_1$, the coherence (7) is important and the more intense radiation is more strongly absorbed, so that the atoms provide negative feedback, which tends to equalize the fields. This has been confirmed numerically and, for example, with initial polarization $\xi = 2$, we find $X_+ \approx X_-$ until the atoms become thoroughly saturated.

PHYSICAL RESTRICTIONS

The collision rates Γ_1 and Γ_2 are related by physical considerations. In terms of Zeeman rates $K_{1,0}$ (for transfer between $m = 0$ and $|m| = 1$) and $K_{1,-1}$ we have $\Gamma_2 = 3K_{1,0}$ and $\Gamma_1 = K_{1,0} + 2K_{1,-1}$, so that immediately $\beta_1 < 3\beta_2$. It can be shown that under the latter condition, the bifurcations occur in the physically inaccessible region between the symmetric branch turning points. Nature is even more restrictive however: experiment shows that $\beta_2 \approx 1.1\beta_1$, and this is supported by detailed collision calculations (Berman & Lamb 1969). This means that a $J_l = 1$, $J_u = 0$, atom in an isotropic collision environment will *never* allow asymmetric output. It is interesting that for $\beta_2 = \beta_1$, then $\eta_+ = \eta_-$, and the system becomes extremely stable in that the input polarization is exactly preserved in the output, i.e. $Y_+/Y_- = X_+/X_-$.

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